

MATHEMATICS

SECTION - I
Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

21. Let $P(3, 2, 6)$ be a point in space and Q be a point on the line $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$. Then the value of μ for which the vector \vec{PQ} is parallel to the plane $x - 4y + 3z = 1$ is

- (A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$

Sol. (A) Given $\vec{OQ} = (1 - 3\mu)\hat{i} + (\mu - 1)\hat{j} + (5\mu + 2)\hat{k}$, $\vec{OP} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ (where O is origin)

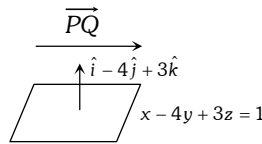
$$\vec{PQ} = (1 - 3\mu - 3)\hat{i} + (\mu - 1 - 2)\hat{j} + (5\mu + 2 - 6)\hat{k}$$

$$= (-2 - 3\mu)\hat{i} + (\mu - 3)\hat{j} + (5\mu - 4)\hat{k}$$

$\therefore \vec{PQ}$ is parallel to the plane $x - 4y + 3z = 1$

$$\therefore -2 - 3\mu - 4\mu + 12 + 15\mu - 12 = 0$$

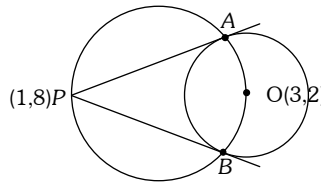
$$\Rightarrow 8\mu = 2 \Rightarrow \mu = \frac{1}{4}$$



22. Tangents drawn from the point $P(1, 8)$ to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B . The equation of the circumcircle of the triangle PAB is

- (A) $x^2 + y^2 + 4x - 6y + 19 = 0$ (B) $x^2 + y^2 - 4x - 10y + 19 = 0$ (C) $x^2 + y^2 - 2x + 6y - 29 = 0$ (D) $x^2 + y^2 - 6x - 4y + 19 = 0$

Sol. (B) For required circle, $P(1, 8)$ and $O(3, 2)$ will be the end points of its diameter.



$$\therefore (x - 1)(x - 3) + (y - 8)(y - 2) = 0$$

$$\Rightarrow x^2 + y^2 - 4x - 10y + 19 = 0$$

23. Let f be a non-negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt$, $0 \leq x \leq 1$, and $f(0) = 0$, then

- (A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$ (B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$ (C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$ (D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

Sol. (C) Given $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt$, $0 \leq x \leq 1$

Apply Leibnitz theorem, we get $\sqrt{1 - (f'(x))^2} = f(x)$

$$\Rightarrow 1 - (f'(x))^2 = f^2(x) \Rightarrow (f'(x))^2 = 1 - f^2(x)$$

$$\Rightarrow f'(x) = \pm \sqrt{1 - f^2(x)} \Rightarrow \frac{dy}{dx} = \pm \sqrt{1 - y^2}, \text{ where } y = f(x)$$

$$\Rightarrow \frac{dy}{\sqrt{1 - y^2}} = \pm dx$$

Integrating both sides

$$\sin^{-1}(y) = \pm x + c$$

$$\therefore f(0) = 0 \Rightarrow c = 0$$

$$\therefore y = \pm \sin x$$

$$y = \sin x = f(x) \text{ given } f(x) \geq 0 \text{ for } x \in [0, 1]$$

It is known that $\sin x < x$, $\forall x \in R^+$

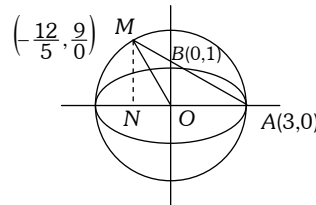
$$\therefore \sin\left(\frac{1}{2}\right) < \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) < \frac{1}{2} \text{ and } \sin\left(\frac{1}{3}\right) < \frac{1}{3} \Rightarrow f\left(\frac{1}{3}\right) < \frac{1}{3}$$

- 24.** Let $z = x + iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $zz^3 + \bar{z}z^3 = 350$ is
 (A) 48 (B) 32 (C) 40 (D) 80

Sol. (A) $z\bar{z}(z^2 + \bar{z}^2) = 350$
 $\Rightarrow 2(x^2 + y^2)(x^2 - y^2) = 350$
 $\Rightarrow (x^2 + y^2)(x^2 - y^2) = 175$
 Since $x, y \in I$, the only possible case which gives integral solution, is
 $x^2 + y^2 = 25$ (i)
 $x^2 - y^2 = 7$ (ii)
 From (i) and (ii), $x^2 = 16$, $y^2 = 9$
 $\Rightarrow x = \pm 4$, $y = \pm 3 \Rightarrow \text{Area} = 48$.

- 25.** The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M . Then the area of the triangle with vertices at A , M and the origin O is
 (A) $\frac{31}{10}$ (B) $\frac{29}{10}$ (C) $\frac{21}{10}$ (D) $\frac{27}{10}$

Sol. (D) Equation of auxiliary circle is $x^2 + y^2 = 9$ (i)
 Equation of AM is $\frac{x}{3} + \frac{y}{1} = 1$ (ii)



On solving (i) and (ii), we get $M\left(-\frac{12}{5}, \frac{9}{5}\right)$
 Now, area of $\triangle AOM = \frac{1}{2} \cdot OA \times MN = \frac{27}{10}$ square unit

- 26.** If $\vec{a}, \vec{b}, \vec{c}$ and \vec{d} are unit vector such that $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$ and $\vec{a} \cdot \vec{c} = \frac{1}{2}$, then
 (A) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar (B) $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar
 (C) \vec{b}, \vec{d} are non-parallel (D) \vec{a}, \vec{d} are parallel and \vec{b}, \vec{c} are parallel

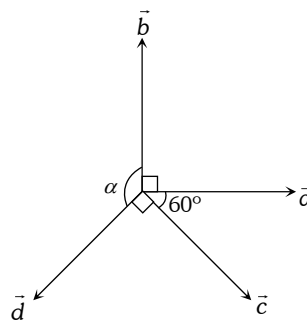
Sol. (C) Let angle between \vec{a} and \vec{b} be θ_1 , \vec{c} and \vec{d} be θ_2 and $\vec{a} \times \vec{b}$ and $\vec{c} \times \vec{d}$ be θ .

$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$
 $\Rightarrow \sin \theta_1 \cdot \sin \theta_2 \cdot \cos \theta = 1$
 $\Rightarrow \theta_1 = 90^\circ, \theta_2 = 90^\circ, \theta = 0^\circ$
 $\Rightarrow \vec{a} \perp \vec{b}, \vec{c} \perp \vec{d}, (\vec{a} \times \vec{b}) \parallel (\vec{c} \times \vec{d})$
 so $\vec{a} \times \vec{b} = k(\vec{c} \times \vec{d})$ and $\vec{a} \times \vec{b} = k(\vec{c} \times \vec{d})$
 $\Rightarrow (\vec{a} \times \vec{b}) \cdot \vec{c} = k(\vec{c} \times \vec{d}) \cdot \vec{c}$ and $(\vec{a} \times \vec{b}) \cdot \vec{d} = k(\vec{c} \times \vec{d}) \cdot \vec{d}$
 $\Rightarrow [\vec{a} \vec{b} \vec{c}] = 0$ and $[\vec{a} \vec{b} \vec{d}] = 0$
 $\Rightarrow \vec{a}, \vec{b}, \vec{c}$ and $\vec{a}, \vec{b}, \vec{d}$ are coplanar vectors. So options A and B are incorrect
 $\Rightarrow \text{Let } \vec{b} \parallel \vec{d} \Rightarrow \vec{b} = \pm \vec{d}$

as $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1 \Rightarrow (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{b}) = \pm 1$
 $\Rightarrow [\vec{a} \times \vec{b} \vec{c} \vec{b}] = \pm 1$
 $\Rightarrow [\vec{c} \vec{b} \vec{a} \times \vec{b}] = \pm 1$
 $\Rightarrow \vec{c} \cdot [\vec{b} \times (\vec{a} \times \vec{b})] = \pm 1$
 $\Rightarrow \vec{c} \cdot [\vec{a} - (\vec{b} \cdot \vec{a})\vec{b}] = \pm 1$
 $\Rightarrow \vec{c} \cdot \vec{a} = \pm 1$ [$\because \vec{a} \cdot \vec{b} = 0$]

which is a contradiction so option C is correct
 Let option D is correct
 $\Rightarrow \vec{d} = \pm \vec{a}$ and $\vec{c} = \pm \vec{b}$

as $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$
 $\Rightarrow (\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{a}) = \pm 1$
 which is a contradiction so option D is incorrect



Alternatively option C and D may be observed from the given figure.

27. Let $z = \cos \theta + i \sin \theta$. Then the value of $\sum_{m=1}^{15} \operatorname{Im}(z^{2m-1})$ at $\theta = 2^\circ$ is

- (A) $\frac{1}{\sin 2^\circ}$ (B) $\frac{1}{3 \sin 2^\circ}$ (C) $\frac{1}{2 \sin 2^\circ}$ (D) $\frac{1}{4 \sin 2^\circ}$

Sol. (D) Given that $z = \cos \theta + i \sin \theta = e^{i\theta}$

$$\begin{aligned} \therefore \sum_{m=1}^{15} \operatorname{Im}(z^{2m-1}) &= \sum_{m=1}^{15} \operatorname{Im}(e^{i\theta})^{2m-1} = \sum_{m=1}^{15} \operatorname{Im} e^{i(2m-1)\theta} = \sin \theta + \sin 3\theta + \sin 5\theta + \dots + \sin 29\theta \\ &= \frac{\sin\left(\frac{\theta + 29\theta}{2}\right) \sin\left(\frac{15 \times 2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)} = \frac{\sin(15\theta) \sin(15\theta)}{\sin \theta} = \frac{1}{4 \sin 2^\circ} \end{aligned}$$

28. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits, 1, 2 and 3 only, is

- (A) 55 (B) 66 (C) 77 (D) 88

Sol. (C) There are two possible cases

Case I : Five 1's, one 2's, one 3's

$$\text{Number of numbers} = \frac{7!}{5!} = 42$$

Case II : Four 1's, three 2's

$$\text{Number of numbers} = \frac{7!}{4!3!} = 35$$

$$\text{Total number of numbers} = 42 + 35 = 77.$$

SECTION - II Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

29. Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is

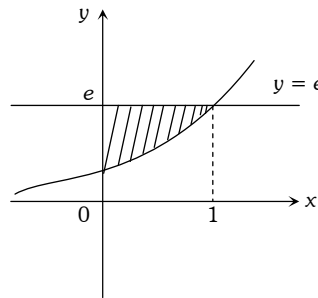
- (A) $e - 1$ (B) $\int_1^e \ln(e + 1 - y) dy$ (C) $e - \int_0^1 e^x dx$ (D) $\int_1^e \ln y dy$

Sol. (B,C,D) Shaded area $= e - \left(\int_0^1 e^x dx \right) = 1$

$$\text{Also, } \int_1^e \ln(e + 1 - y) dy$$

$$\text{put } e + 1 - y = t \Rightarrow -dy = dt$$

$$= \int_e^1 \ln t (-dt) = \int_1^e \ln t dt = \int_1^e \ln y dy = 1$$



30. Let $L = \lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then

- (A) $a = 2$ (B) $a = 1$ (C) $L = \frac{1}{64}$ (D) $L = \frac{1}{32}$

$$\begin{aligned} \text{Sol. (A,C)} \quad L &= \lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0 = \lim_{x \rightarrow 0} \frac{a - a \left[1 - \frac{1}{2} \cdot \frac{x^2}{a^2} + \frac{1}{2} \left(\frac{1}{2} - 1 \right) \cdot \frac{x^4}{a^4} - \dots \right] - \frac{x^2}{4}}{x^4} \\ &= \lim_{x \rightarrow 0} \frac{\frac{x^2}{2a} + \frac{1}{8} \cdot \frac{x^4}{a^3} + \dots - \frac{x^2}{4}}{x^4} \end{aligned}$$

Since, L is finite $\Rightarrow 2a = 4 \Rightarrow a = 2$

$$\therefore L = \lim_{x \rightarrow 0} \frac{1}{8a^3} = \frac{1}{64}$$

- 31.** In a triangle ABC with fixed base BC , the vertex A moves such that $\cos B + \cos C = 4 \sin^2 \frac{A}{2}$. If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C , respectively, then
- (A) $b + c = 4a$ (B) $b + c = 2a$
(C) Locus of point A is an ellipse (D) Locus of point A is a pair of straight lines

Sol. (B,C) $\cos B + \cos C = 4 \sin^2 \frac{A}{2}$

$$\Rightarrow 2 \cos \frac{B+C}{2} \cos \frac{B-C}{2} = 4 \sin^2 \frac{A}{2}$$

$$\Rightarrow 2 \sin \frac{A}{2} \left[\cos \frac{B-C}{2} - 2 \sin \frac{A}{2} \right] = 0$$

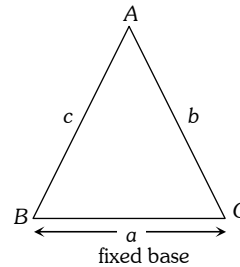
$$\Rightarrow \cos \left(\frac{B-C}{2} \right) - 2 \cos \left(\frac{B+C}{2} \right) = 0 \quad \text{as } \sin \frac{A}{2} \neq 0$$

$$\Rightarrow -\cos \frac{B}{2} \cos \frac{C}{2} + 3 \sin \frac{B}{2} \sin \frac{C}{2} = 0$$

$$\Rightarrow \tan \frac{B}{2} \tan \frac{C}{2} = \frac{1}{3}$$

$$\Rightarrow \sqrt{\frac{(s-a)(s-c)}{s(s-b)} \cdot \frac{(s-b)(s-a)}{s(s-c)}} = \frac{1}{3}$$

$$\Rightarrow \frac{s-a}{s} = \frac{1}{3} \Rightarrow 2s = 3a \Rightarrow b + c = 2a$$



\therefore Locus of point A is an ellipse.

- 32.** If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then
- (A) $\tan^2 x = \frac{2}{3}$ (B) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$ (C) $\tan^2 x = \frac{1}{3}$ (D) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

Sol. (A,B) $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5} \Rightarrow \frac{\sin^4 x}{2} + \frac{(1 - \sin^2 x)^2}{3} = \frac{1}{5}$

$$\Rightarrow \frac{\sin^4 x}{2} + \frac{1 + \sin^4 x - 2 \sin^2 x}{3} = \frac{1}{5}$$

$$\Rightarrow 5 \sin^4 x - 4 \sin^2 x + 2 = \frac{6}{5}$$

$$\Rightarrow 25 \sin^4 x - 20 \sin^2 x + 4 = 0$$

$$\Rightarrow (5 \sin^2 x - 2)^2 = 0$$

$$\Rightarrow \sin^2 x = \frac{2}{5}, \cos^2 x = \frac{3}{5}$$

$$\Rightarrow \tan^2 x = \frac{2}{3}$$

and $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$.

SECTION - III Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 33 to 35

Let \mathcal{A} be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

- 33.** The number of matrices in \mathcal{A} is
- (A) 12 (B) 6 (C) 9 (D) 3
- Sol. (A) Case I :** All three diagonal elements are 1
No. of matrices = ${}^3C_1 = 3$

Case II : Two diagonal elements are zero and one element is one

$$\text{No. of matrices} = {}^3C_1 \cdot {}^3C_1 = 9$$

$$\text{Total matrices} = 3 + 9 = 12.$$

34. The number of matrices A in \mathcal{A} for which the system of linear equations $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution, is

- (A) Less than 4 (B) At least 4 but less than 7 (C) At least 7 but less than 10 (D) At least 10

Sol. (B) $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

For unique solution $\det(A) \neq 0$

Case I : $\det(A) = \begin{vmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{vmatrix} = 1 - a^2 - b^2 - c^2 + 2abc \neq 0$

Here a, b, c is selected from 1, 0, 0. (No case is possible)

Case II :

(i) $\det(A) = \begin{vmatrix} 1 & a & b \\ a & 0 & c \\ b & c & 0 \end{vmatrix} = 2abc - c^2 \neq 0$

Here a, b, c are selected from 1, 1, 0. (2 cases are possible)

(ii) $\det(A) = \begin{vmatrix} 0 & a & b \\ a & 1 & c \\ b & c & 0 \end{vmatrix} = 2abc - b^2 \neq 0$

Here a, b, c are selected from 1, 1, 0. (2 cases are possible)

(iii) $\det(A) = \begin{vmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 1 \end{vmatrix} = 2abc - a^2 \neq 0$

Here a, b, c are selected from 1, 1, 0. (2 cases are possible)

Hence, there are exactly 6 matrices for unique solution. Hence option B is correct.

35. The number of matrices A in \mathcal{A} for which the system of linear equations $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent, is

- (A) 0 (B) More than 2 (C) 2 (D) 1

Sol. (B) $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Case I : $\begin{bmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

a, b, c are selected from 1, 0, 0.

$$\Rightarrow x + ay + bz = 1$$

$$ax + y + cz = 0$$

$$bx + cy + z = 0$$

(i) If $a = 1, b = c = 0$ then $x + y = 1$. Inconsistent system of equation
 $x + y = 0$

(ii) If $a = 0 = c, b = 1$ then $x + z = 1$
 $y = 0$. Inconsistent system of equation
 $x + z = 0$

(iii) If $c = 1, a = b = 0$ then $x = 1, z = 0, y = 0$

Case II :

(i) $\begin{bmatrix} 1 & a & b \\ a & 0 & c \\ b & c & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

a, b, c are selected from 1, 1, 0

$$\Rightarrow x + ay + bz = 1$$

$$ax + cz = 0$$

$bx + cy = 0$. Clearly, In all three cases, solutions are possible so system is consistent.

$$(ii) \begin{bmatrix} 0 & a & b \\ a & 1 & c \\ b & c & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow ay + bz = 1$$

$$ax + y + cz = 0$$

$$bx + cy = 0$$

Clearly, $b = 0$, $a = c = 1$ gives

$$y = 1$$

$$x + y + z = 0 \text{ . Inconsistent system}$$

$$y = 0$$

More than 2 matrices are possible. Hence option B is correct

Paragraph for Question Nos. 36 to 38

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

36. The probability that $X=3$ equals

- (A) $\frac{25}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{125}{216}$

Sol. (A) $P(X=3) = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} = \frac{25}{216}$.

37. The probability that $X \geq 3$ equals

- (A) $\frac{125}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{25}{216}$

Sol. (B) $P(X \geq 3) = \frac{5}{6} \cdot \frac{5}{6} \cdot 1 = \frac{25}{36}$.

38. The conditional probability that $X \geq 6$ given $X > 3$ equals

- (A) $\frac{125}{216}$ (B) $\frac{25}{216}$ (C) $\frac{5}{36}$ (D) $\frac{25}{36}$

Sol. (D) $P((X \geq 6)/(X > 3)) = \frac{P((X > 3)/(X \geq 6)) \cdot P(X \geq 6)}{P(X > 3)} = \frac{1 \left[\left(\frac{5}{6}\right)^5 \cdot \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^6 \cdot \left(\frac{1}{6}\right) + \dots \infty \right]}{\left[\left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \dots \infty \right]} = \frac{25}{36}$.

SECTION - IV Matrix - Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A- p, s and t; B - q and r; C - p and q; and D - s and t; then the correct darkening of bubbles will look like the following :

	p	q	r	s	t
A	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>
B	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
C	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
D	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>

39. Match the statements/expressions in **Column I** with the open intervals in **Column II**.

Column I

- (A) Interval contained in the domain of definition of non-zero solutions of the differential equation $(x-3)^2 y' + y = 0$ (p)
- (B) Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5) dx$ (q)
- (C) Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies (r)
- (D) Interval in which $\tan^{-1}(\sin x + \cos x)$ is increasing (s)
- (t)

Column II

- $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- $\left(0, \frac{\pi}{2}\right)$
- $\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$
- $\left(0, \frac{\pi}{8}\right)$
- $(-\pi, \pi)$

Sol. (A) $(x-3)^2 \cdot y' + y = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{(x-3)^2} \Rightarrow \int \frac{dy}{y} = -\int \frac{dx}{(x-3)^2} \Rightarrow \ln y = \frac{1}{x-3} + \ln C \Rightarrow y = Ce^{\frac{1}{x-3}}, C \neq 0$$

$x \in \mathbb{R} - \{3\}$. Hence correct options p, q, s.

Aliter : Given differential equation is homogeneous linear differential equation and has $x=3$ as a singular point hence $x=3$ cannot be in domain of solution.

(B) Let $I = \int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5) dx$

Let $x-3=t \Rightarrow dx=dt$

$$\therefore I = \int_{-2}^2 (t+2)(t+1)t(t-1)(t-2) dt$$

\therefore Integrand is an odd function

$\therefore I = 0$. Hence correct options are p, t.

Aliter : Let $I = \int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5) dx$ (i)

Using $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

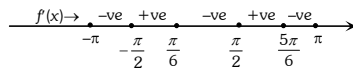
$$\therefore I = \int_1^5 (5-x)(4-x)(3-x)(2-x)(1-x) dx \quad \text{.....(ii)}$$

On adding (i) and (ii), we get $2I = 0 \Rightarrow I = 0$.

(C) $f(x) = \cos^2 x + \sin x$

$f'(x) = -2 \cos x \sin x + \cos x = \cos x(1 - 2 \sin x) = 0$. Hence correct options are p, t.

Sign scheme for first derivative



Points of local maxima are $\frac{\pi}{6}, \frac{5\pi}{6}$. Clearly option (p, q, r, t) are correct.

Aliter : $y = \cos^2 x + \sin x$

$$y = \frac{5}{4} - \left(\sin x - \frac{1}{2}\right)^2$$

For y to be maximum

$$\left(\sin x - \frac{1}{2}\right)^2 = 0$$

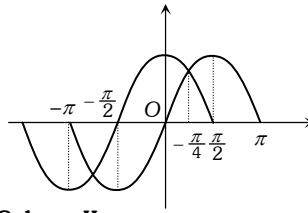
$$\Rightarrow \sin x = \frac{1}{2}$$

$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{6}, n \in I$. Hence option (p, q, r, t) are correct.

(D) $y = \tan^{-1}(\sin x + \cos x)$

$$\frac{dy}{dx} = \frac{\cos x - \sin x}{1 + (\sin x + \cos x)^2}$$

Clearly by graph $\cos x > \sin x$ is true for options (s)



(A) \rightarrow p, q, s; (B) \rightarrow p, t; (C) \rightarrow p, q, r, t; (D) \rightarrow s.

40. Match the conics in **Column I** with the statements/expressions in **Column II**.

Column I

- (A) Circle
- (B) Parabola
- (C) Ellipse
- (D) Hyperbola

Column II

- (p) The locus of the point (h, k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$
- (q) Points z in the complex plane satisfying $|z + 2| - |z - 2| = \pm 3$
- (r) Points of the conic have parametric representation $x = \sqrt{3} \left(\frac{1-t^2}{1+t^2} \right), y = \frac{2t}{1+t^2}$
- (s) The eccentricity of the conic lies in the interval $1 \leq e < \infty$
- (t) Points z in the complex plane satisfying $\operatorname{Re}(z+1)^2 = |z|^2 + 1$

Sol. (p) $\frac{1}{\sqrt{h^2 + k^2}} = 2 \Rightarrow h^2 + k^2 = \frac{1}{4} \Rightarrow$ locus is a circle.

(q) $||z + 2| - |z - 2|| = 3$ and $2 - (-2) = 4 > 3 \Rightarrow$ locus is a hyperbola.

(r) $x = \sqrt{3} \left(\frac{1-t^2}{1+t^2} \right), y = \frac{2t}{1+t^2}$

Let $\tan \theta = t$

$x = \sqrt{3} \cos 2\theta, y = \sin 2\theta$

Hence, $\frac{x^2}{3} + y^2 = 1 \Rightarrow$ locus is an ellipse.

(s) Eccentricity $e = 1 \Rightarrow$ Parabola, $1 < e < \infty \Rightarrow$ Hyperbola.

(t) $z = x + iy$

$\operatorname{Re}(z+1)^2 = |z|^2 + 1$

$\Rightarrow (x+1)^2 - y^2 = x^2 + y^2 + 1$

$\Rightarrow 2x = 2y^2$

$\Rightarrow x = y^2$. Hence locus is a parabola.

(A) \rightarrow p; (B) \rightarrow s, t; (C) \rightarrow r; (D) \rightarrow q, s.