PHYSICS

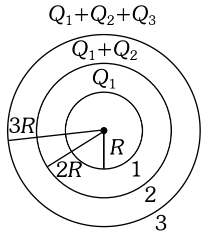
PART – III
SECTION – I
Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

41. Three concentric metallic spherical shells of radii R, $2R$, $3R$, are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is

(A) 1 : 2 : 3 (B) 1 : 3 : 5 (C) 1 : 4 : 9 (D) 1 : 8 : 18

Sol. **(B)** Due to induction net charges on outer surfaces of spheres are as shown.



$$\sigma = \frac{Q_1}{4\pi R^2} = \frac{Q_1+Q_2}{4\pi(2R)^2} = \frac{Q_1+Q_2+Q_3}{4\pi(3R)^2}$$

$$\Rightarrow Q_1 = \frac{Q_1+Q_2}{4} = \frac{Q_1+Q_2+Q_3}{9}$$

$$\Rightarrow Q_2 = 3Q_1 \text{ and } Q_3 = 5Q_1$$

$$\therefore Q_1 : Q_2 : Q_3 = 1:3:5.$$

42. A block of base $10\ cm \times 10 cm$ and height $15\ cm$ is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from $0°$. Then

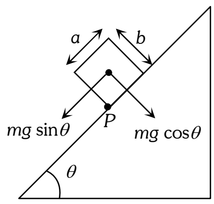
(A) At $\theta = 30°$, the block will start sliding down the plane

(B) The block will remain at rest on the plane up to certain θ and then it will topple

(C) At $\theta = 60°$, the block will start sliding down the plane and continue to do so at higher angles

(D) At $\theta = 60°$, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

Sol. **(B)** For rotational equilibrium about point "P",



$$mg\sin\theta\left(\frac{b}{2}\right) = mg\cos\theta\left(\frac{a}{2}\right)$$

$$\Rightarrow \tan\theta = \frac{a}{b} = \frac{10}{15} = \frac{2}{3} \Rightarrow \tan\theta = \frac{2}{3} \Rightarrow \theta = 33.69°$$

i.e., toppling starts at $\theta = 33.69°$

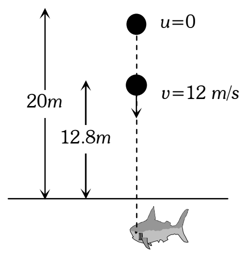
and angle of repose $= \tan^{-1}(\mu) = \tan^{-1}(\sqrt{3}) = 60°$

it mean the block will remain at rest on the plane up to certain θ and then it will topple.

43. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as

(A) 9 m/s (B) 12 m/s (C) 16 m/s (D) 21.33 m/s

Sol. **(C)** Velocity of the ball when it reaches at the height of 12.8 m.



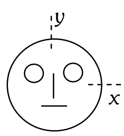
Above the surface is $v = \sqrt{2 \times 10 \times 12.8} = 12\ m/s$

Height of the ball from surface as seen by fish

$$h' = \mu h \Rightarrow \frac{dh'}{dt} = \mu\frac{dh}{dt}$$

$$\Rightarrow v' = \mu v = \frac{4}{3} \times 12 = 16\ m/s.$$

44. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is 6 m. The coordinates of the centres of the different parts are : outer circle (0, 0), left inner circle (–a, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0, –a). The y-coordinate of the centre of mass of the ink in this drawing is



(A) $\frac{a}{10}$ (B) $\frac{a}{8}$ (C) $\frac{a}{12}$ (D) $\frac{a}{3}$

Sol. **(A)** According to problem $m_1 = 6m, m_2 = m_3 = m_4 = m_5 = m$

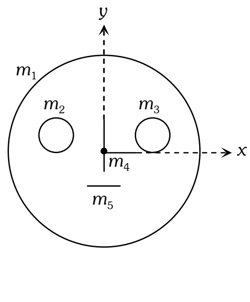
$\vec{r_1} = 0\hat{i} + 0\hat{j}$, $\vec{r_2} = -a\hat{i} + a\hat{j}$, $\vec{r_3} = a\hat{i} + a\hat{j}$, $\vec{r_4} = 0\hat{i} + 0\hat{j}$, $\vec{r_5} = 0\hat{i} - a\hat{j}$

Position vector of centre of mass

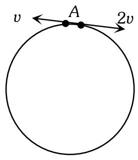
$$\vec{r_{cm}} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3} + m_4\vec{r_4} + m_5\vec{r_5}}{m_1 + m_2 + m_3 + m_4 + m_5}$$

$$\vec{r_{cm}} = \frac{0 + m(-a\hat{i} + a\hat{j}) + m(a\hat{i} + a\hat{j}) + 0 + m(-a\hat{j})}{10m} = 0\hat{i} + \frac{a}{10}\hat{j}$$

So, the coordinate of centre of mass $= \left(0, \frac{a}{10}\right)$.



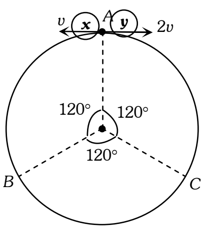
45. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A

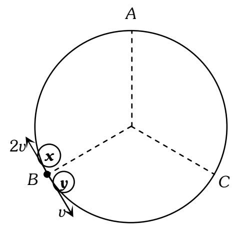


(A) 4 (B) 3 (C) 2 (D) 1

Sol. **(C)** Let initially particle x is moving in anticlockwise direction and y in anticlockwise direction.

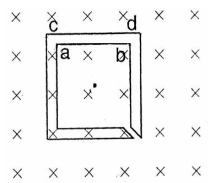
As the ratio of velocities of x and y particles are $\frac{v_x}{v_y} = \frac{1}{2}$, therefore ratio of their distance covered will be in the ratio of 2 : 1. It means they collide at point B.





After first collision at B, velocities of particles get interchanged, *i.e.*, x will move with $2v$ and particle y with v.
Second collision will take place at point C. Again at this point velocities get interchanged and third collision take place at point A.
So, after two collision these two particles will again reach the point A.

46. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments **ab** and **cd.** Then,

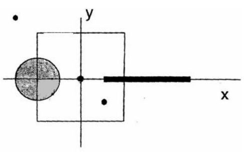


(A) $I_1 > I_2$ (B) $I_1 < I_2$

(C) I_1 is in the direction **ba** and I_2 is in the direction **cd** (D) I_1 is in the direction **ab** and I_2 is in the direction **dc**

Sol. **(D)** Due to decrease in crosses (X), induced current in outer loop is anticlockwise, *i.e.*, from d to c and clockwise in inner loop *i.e.,* from $a \to b$.

47. A disk of radius a/4 having a uniformly distributed charge 6C is placed in the x-y plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge 8C is placed on the x-axis from $x = a/4$ to $x = 5a/4$. Two point charges $-7C$ and $3C$ are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2, y = \pm a/2, z = \pm a/2$. The electric flux through this cubical surface is

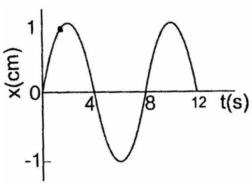


(A) $\frac{-2C}{\varepsilon_0}$ (B) $\frac{2C}{\varepsilon_0}$ (C) $\frac{10C}{\varepsilon_0}$ (D) $\frac{12C}{\varepsilon_0}$

Sol. **(A)** Half disk (charge 3C), are fourth rod (charge 2C) and charge of –7C are inside the cubical surface, so net charge inside the surface $= 3 + 2 - 7 = -2C$

$\therefore$ Flux through the surface $\phi = \frac{1}{\varepsilon_0}(Q) = \frac{-2C}{\varepsilon_0}$.

48. The *x-t* graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3\ s$ is



(A) $\frac{\sqrt{3}}{32}\pi^2\ cm/s^2$ (B) $\frac{-\pi^2}{32}\ cm/s^2$ (C) $\frac{\pi^2}{32}\ cm/s^2$ (D) $-\frac{\sqrt{3}}{32}\pi^2\ cm/s^2$

Sol. **(D)** From given graph amplitude $(a) = 1cm$

Time period $(T) = 8\ sec$

$\therefore\ \omega = \frac{2\pi}{8} = \frac{\pi}{4}\ Hz$

Acceleration $A = -\omega^2 a \sin \omega t$

at $t = \frac{4}{3}$ sec, $A = -\frac{\pi^2}{16} \times 1 \times \sin\left(\frac{\pi}{4} \times \frac{4}{3}\right) \Rightarrow A = \frac{-\pi^2}{16}\sin\left(\frac{\pi}{3}\right) \Rightarrow A = \frac{-\sqrt{3}}{32}\pi^2\ cm/s^2$.

SECTION – II

Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

49. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that

(A) Linear momentum of the system does not change in time
(B) Kinetic energy of the system does not change in time
(C) Angular momentum of the system does not change in time
(D) Potential energy of the system does not change in time

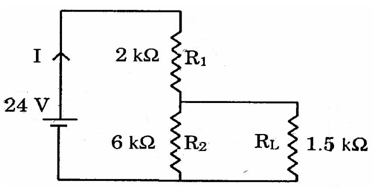
Sol. **(A)**

50. A student performed the experiment of determination of focal length of a concave mirror by u-v method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 *cm*. The maximum error in the location of the image can be 0.2 *cm*. The 5 sets of (u, v) values recorded by the student (in *cm*) are : (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that **cannot** come from experiment and is (are) incorrectly recorded, is (are)

(A) (42, 56) (B) (48, 48) (C) (66, 33) (D) (78, 39)

Sol. **(C,D)**

51. For the circuit shown in the figure



(A) The current *I* through the battery is 7.5 *mA*
(B) The potential difference across R_L is 18 *V*
(C) Ratio of powers dissipated in R_1 and R_2 is 3
(D) If R_1 and R_2 are interchanged, magnitude of the power dissipated in R_L will decrease by a factor of 9

Sol. **(A,D)** Equivalent resistance $R_{eq} = 2 + \frac{6 \times 1.5}{(6+1.5)} = 3.2\ k\Omega$

$\therefore$ Current $i = \frac{24}{3.2 \times 10^3} = 7.5\ mA$

Potential difference across the parallel combination of R_L & R_L = Potential difference across $R_L = 7.5 \times 10^{-3} \times \left(\frac{6 \times 1.5}{6+1.5}\right) \times 10^3 = 9V$

Ratio of power dissipated in R_1 & R_2 is

$\frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^2 \times \frac{R_2}{R_1}$ $[\because P = V^2/R]$

$= \left(\frac{15}{9}\right)^2 \times \frac{6}{2} = \frac{25}{3}$

when R_1 & R_2 interchanged then potential difference across R_L change from 9V to 3V.

According to $P = V^2/R$, power dissipated decreases by a factor of 9.

52. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then

(A) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(B) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(C) C_p / C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas

(D) $C_p . C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

Sol. **(B,D)** For monoatomic gas $C_v = \frac{3}{2}R,\ \ C_p = \frac{5}{2}R$

and for diatomic gas $C_v = \frac{5}{2}R,\ \ C_p = \frac{7}{2}R$

$C_p - C_v = R$ (for all gases). So option (A) is not correct.

$C_p + C_v = 4R$ (for monoatomic gases) $= 6R$ (for diatomic gases). So option (B) is correct.

$\frac{C_p}{C_v} = \frac{5}{3} = 1.67$ (for monoatomic gases) $= \frac{7}{5} = 1.4$ (for diatomic gases). So option (C) is incorrect.

$C_p \times C_v = \frac{15}{4}R^2$ (for monoatomic gases) $= \frac{35}{4}R^2$ (for diatomic gases). So option (D) is correct.

SECTION – III
Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 53 to 55

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}_1^2H$, known as deuteron and denoted by *D*, can be thought of as a candidate for fusion reactor. The *D-D* reaction is ${}_1^2H + {}_1^2H \rightarrow {}_2^3He + n +$ energy. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}_1^2H$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If *n* is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than $5 \times 10^{14}\ s/cm^3$.

It may be helpful to use the following : Boltzmann constant $k = 8.6 \times 10^{-5}\ eV/K;\ \frac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}\ eVm$.

53. In the core of nuclear fusion reactor, the gas becomes plasma because of

(A) Strong nuclear force acting between the deuterons

(B) Coulomb force acting between the deuterons

(C) Coulomb force acting between deuteron-electron pairs

(D) The high temperature maintained inside the reactor core

Sol. **(D)**

54. Assume that two deuteron nuclei in the core of fusion reactor at temperature *T* are moving towards each other, each with kinetic energy $1.5\ kT$, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature *T* required for them to reach a separation of $4 \times 10^{-15}\ m$ is in the range

(A) $1.0 \times 10^9\ K < T < 2.0 \times 10^9\ K$

(B) $2.0 \times 10^9\ K < T < 3.0 \times 10^9\ K$

(C) $3.0 \times 10^9\ K < T < 4.0 \times 10^9\ K$

(D) $4.0 \times 10^9\ K < T < 5.0 \times 10^9\ K$

Sol. **(B)** Energy corresponding to given separation is

$$E = 9 \times 10^9 \times \frac{e^2}{r} = 9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{(4 \times 10^{-15})} = 5.76 \times 10^{-14}\ J$$

$1.5\ kT = 14.4$

$$\Rightarrow T = \frac{5.76 \times 10^{-14}}{1.5 \times 1.38 \times 10^{-23}} = 2.78 \times 10^9\ K$$

At the temperature $3 \times 10^9\ K$ fusion begins. Hence, temperature range will be $2 \times 10^9\ K < T < 3 \times 10^9\ K$.

55. Results of calculations for four different designs of a fusion reactor using *D-D* reaction are given below. Which of these is most promising based on Lawson criterion?

(A) Deuteron density $= 2.0\times10^{12}\ cm^{-3}$, confinement time $= 5.0\times10^{-3}\ s$

(B) Deuteron density $= 8.0\times10^{14}\ cm^{-3}$, confinement time $= 9.0\times10^{-1}\ s$

(C) Deuteron density $= 4.0\times10^{23}\ cm^{-3}$, confinement time $= 1.0\times10^{-11}\ s$

(D) Deuteron density $= 1.0\times10^{24}\ cm^{-3}$, confinement time $= 4.0\times10^{-12}\ s$

Sol. **(B)** A successful fusion reactor must satisfy Lawson's criterion $n\,\tau > 10^{14}\ s/cm^3$
where n = deuteron density
and τ = long confinement time.
This condition is satisfied only in option (B).

Paragraph for Question Nos. 56 to 58

When a particle is restricted to move along *x*-axis between $x = 0$ and $x = a$, where a is nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number n' taking values 1, 2, 3, ... (n=1, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line $x = 0$ to $x = a$. Take $h = 6.6\times10^{-34}\ Js$ and $e = 1.6\times10^{-19}\ C$.

56. The allowed energy for the particle for a particular value of n is proportional to

(A) a^{-2} (B) $a^{-3/2}$ (C) a^{-1} (D) a^2

Sol. **(A)** $E = \frac{p^2}{2m}$; $p = \frac{h}{\lambda} \Rightarrow E = \frac{h^2}{2m\lambda^2}$

for standing waves $\lambda = \frac{2a}{n}$

$\Rightarrow E = \frac{h^2n^2}{8ma^2} \Rightarrow E \propto a^{-2}$.

57. If the mass of the particle is $m = 1.0\times10^{-30}\ kg$ and $a = 6.6\ nm$, the energy of the particle in its ground state is closest to

(A) 0.8 meV (B) 8 meV (C) 80 meV (D) 800 meV

Sol. **(B)** $E = \frac{(6.6\times10^{-34})^2\times1^2}{8\times(1\times10^{-30})\times(6.6\times10^{-9})^2} = \frac{1}{8}\times10^{-20}\ J$

$E(eV) = \frac{10^{-20}}{8\times1.0\times10^{-19}} = 7.81\times10^{-3}eV \approx 8\ meV$.

58. The speed of the particle, that can take discrete values, is proportional to

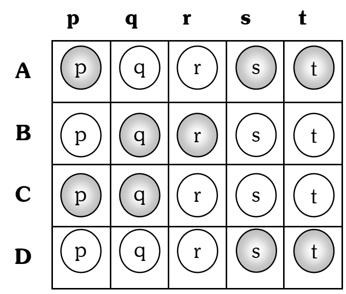
(A) $n^{-3/2}$ (B) n^{-1} (C) $n^{1/2}$ (D) n

Sol. **(D)** $E = \frac{n^2h^2}{8ma^2} = \frac{1}{2}mv^2 \Rightarrow v = \frac{nh}{2ma} \Rightarrow v \propto n$.

SECTION – IV
Matrix – Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled, A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

It the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following.



59. Six point charges, each of the same magnitude q, are arranged in different manners as shown in **Column II**. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotation charge to be equivalent to a steady current.

Column I

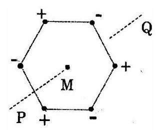
(A) $E = 0$

(B) $V \neq 0$

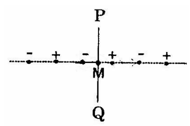
(C) $B = 0$

(D) $\mu \neq 0$

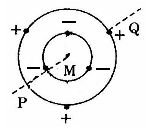
Column II

(p) 

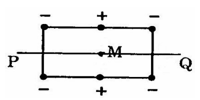
Charge are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon.

(q) 

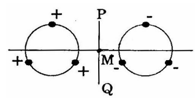
Charges are on a line perpendicular to PQ at equal intervals. M is the mid-point between the two innermost charges.

(r) 

Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings.

(s) 

Charges are placed at the corners of a rectangle of sides a and $2a$ and at the mid points of the longer sides. M is at the centre of the rectangle. PQ is parallel to the longer sides.

(t) 

Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid-point between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar to the rings.

Sol. **(A → p, r, s); (B → r, s, t); (C → p, q, t); (D → r, s).**

60. **Column II** shows five systems in which two objects are labelled as X and Y. Also in each case a point P is shown. **Column I** gives some statements about X and/or Y. Match these statements to the appropriate system(s) from **Column II.**

Column I

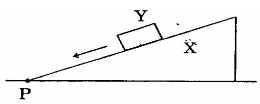
(A) The force exerted by X on Y has a magnitude Mg.

(B) The gravitational potential energy of X is continuously increasing.

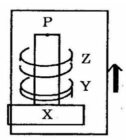
(C) Mechanical energy of the system $X+Y$ is continuously decreasing.

(D) The torque of the weight of Y about point P is zero.

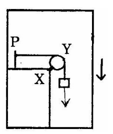
Column II

(p) 

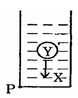
Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity.

(q) 

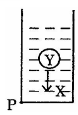
Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity.

(r) 

A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity.

(s) 

A sphere Y of mass M is put in a nonviscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid.

(t) 

A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.

Sol. **(A → p, t); (B → q); (C → p, r, t); (D → q).**