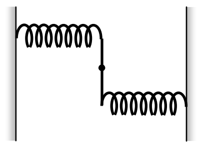
PHYSICS

PART – III
SECTION – I
Single Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

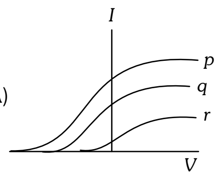
39. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is

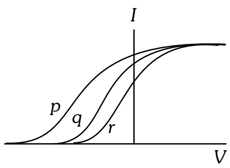


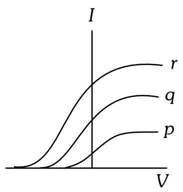
(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

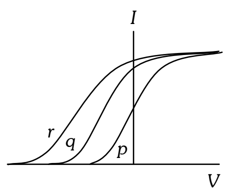
Sol. **(C)**

40. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0\ eV$, $\phi_q = 2.5\ eV$ and $\phi_r = 3.0\ eV$, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is. (Take $hc = 1240\ eV\ nm$]

(A) 

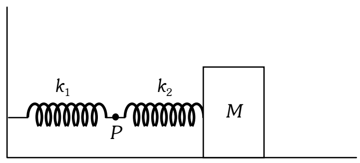
(B) 

(C) 

(D) 

Sol. **(A)**

41. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



(A) $\frac{k_1 A}{k_2}$ (B) $\frac{k_2 A}{k_1}$ (C) $\frac{k_1 A}{k_1 + k_2}$ (D) $\frac{k_2 A}{k_1 + k_2}$

Sol. **(D)** Let the amplitude of spring first is x_1 and of spring second is x_2.

According to problem $x_1 + x_2 = A$ .....(i)

and we know that $\frac{x_1}{x_2} = \frac{k_2}{k_1}$ .....(ii)

from (i) and (ii) $x_1 = \frac{k_2 A}{k_1 + k_2}$

So, the amplitude of point P is $\frac{k_2 A}{k_1 + k_2}$

42. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is

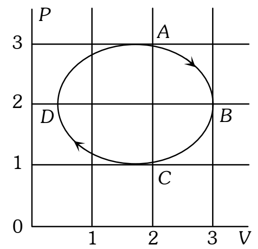
(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$ (C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$

Sol. **(B)**

SECTION – II
Multiple Correct Choice Type

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

43. The figure shows the *P-V* plot of an ideal gas taken through a cycle *ABCDA*. The part *ABC* is a semi-circle and *CDA* is half of an ellipse. Then,



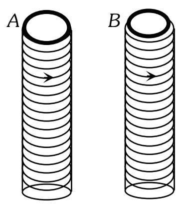
(A) The process during the path A → B is isothermal
(B) Heat flows out of the gas during the path B → C → D
(C) Work done during the path A → B → C is zero
(D) Positive work is done by the gas in the cycle *ABCDA*

Sol. **(B, D)** During path A → B, *PV* = constant i.e., process is isothermal. Option (A) is correct.
During path A → B → C, work done = Area of semicircle (ABC). Option (C) is incorrect.
Cycle ABCDA is moving clockwise it mean positive work is done by the gas. Option (D) is correct.

44. Under the influence of the Coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s).
(A) The angular momentum of the charge $-q$ is constant
(B) The linear momentum of the charge $-q$ is constant
(C) The angular velocity of the charge $-q$ is constant
(D) The linear speed of the charge $-q$ is constant

Sol. **(A)**

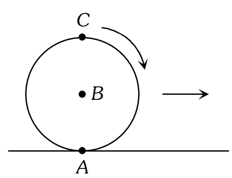
45. Two metallic rings *A* and *B*, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current *I* is switched on in both the solenoids in identical manner, the rings *A* and *B* jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is(are)



(A) $\rho_A > \rho_B$ and $m_A = m_B$
(B) $\rho_A < \rho_B$ and $m_A = m_B$
(C) $\rho_A > \rho_B$ and $m_A > m_B$
(D) $\rho_A < \rho_B$ and $m_A < m_B$

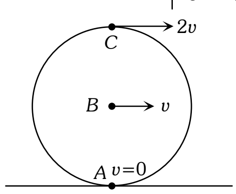
Sol. **(B, D)**

46. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, *A* is the point of contact, *B* is the centre of the sphere and *C* is its topmost point. Then,



(A) $\vec{V}_C - \vec{V}_A = 2\left(\vec{V}_B - \vec{V}_C\right)$
(B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$
(C) $\left|\vec{V}_C - \vec{V}_A\right| = 2\left|\vec{V}_B - \vec{V}_C\right|$
(D) $\left|\vec{V}_C - \vec{V}_A\right| = 4\left|\vec{V}_B\right|$

Sol. **(B, C)** In condition of rolling of sphere without slipping
Velocity of different points of the sphere is shown



$\vec{V}_A = 0, \vec{V}_B = \vec{V}, \vec{V}_C = \overrightarrow{2V}$

$\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A = \vec{V}$. Option (B) is correct.

$|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C| = 2\vec{V}$.

47. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air-column is the second resonance. Then,

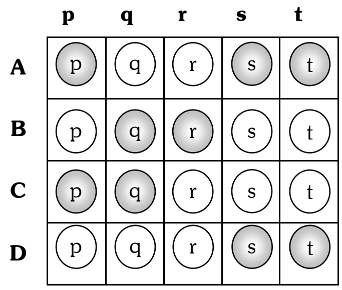
(A) The intensity of the sound heard at the first resonance was more than that at the second resonance
(B) The prongs of the tuning fork were kept in a horizontal plane above the resonance tube
(C) The amplitude of vibration of the ends of the prongs is typically around 1 *cm*
(D) The length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air.

Sol. **(D)**

SECTION – III
Matrix – Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled, A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

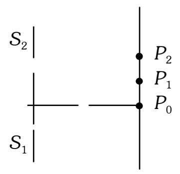
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the following :



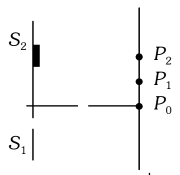
48. **Column I** shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \lambda/4$ and $S_1P_2 - S_2P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in **Column I** with the statement(s) in **Column II** valid for that situation.

Column I

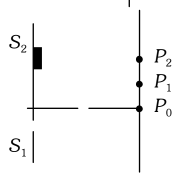
(A)



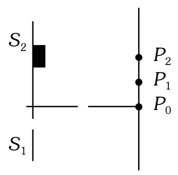
(B) $(\mu - 1)t = \lambda/4$



(C) $(\mu - 1)t = \lambda/2$



(D) $(\mu - 1)t = 3\lambda/4$



Column II

(p) $\delta(P_0) = 0$

(q) $\delta(P_1) = 0$

(r) $I(P_1) = 0$

(s) $I(P_0) > I(P_1)$

(t) $I(P_2) > I(P_1)$

Sol. **(A → p, s) (B → q) (C → t) (D → r, s, t)**

49. **Column II** gives certain systems undergoing a process. **Column I** suggests changes in some of the parameters related to the system. Match the statements in **Column I** to the appropriate process(es) from **Column II.**

Column I

(A) The energy of the system is increased

(B) Mechanical energy is provided to the system, which is converted into energy of random motion of its parts

(C) Internal energy of the system is converted into its mechanical energy

(D) Mass of the system is decreased

Column II

(p) System : A capacitor, initially uncharged
Process : It is connected to a battery

(q) System : A gas in an adiabatic container fitted with an adiabatic piston
Process : The gas is compressed by pushing the piston

(r) System : A gas in a rigid container
Process : The gas gets cooled due to colder atmosphere surrounding it

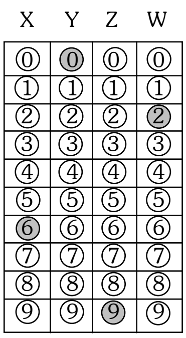
(s) System : A heavy nucleus, initially at rest
Process : The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted

(t) System : A resistive wire loop
Process : The loop is placed in a time varying magnetic field perpendicular to its plane

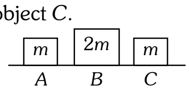
Sol. **(A → p, q, t) (B → q) (C → s) (D → s)**

SECTION – IV
Integer Answer Type

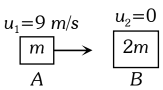
This section contains 8 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following :



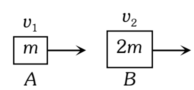
50. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed **(in m/s)** of the object C.



Sol.



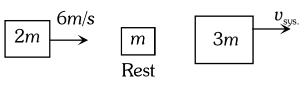
Before elastic collision



After elastic collision

$$v_2 = \frac{2m_1u_1}{m_1+m_2} = \frac{2\times m\times 9}{m+2m} = 6\ m/s$$

i.e., After elastic collision B strikes to C with velocity of 6 m/s. Now collision between B and C is perfectly inelastic.

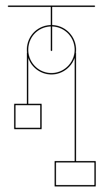


By the law of conservation of momentum

$2m\times 6+0 \ = 3m\times v_{sys}$

$\Rightarrow\ v_{sys} = 4\ m/s$.

51. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 *kg* and 0.72 *kg*. Taking $g = 10\ m/s^2$, find the work done **(in joules)** by the string on the block of mass 0.36 *kg* during the first second after the system is released from rest.



Sol. In the given condition tension in the spring

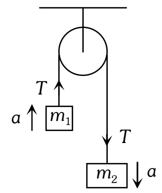
$$T = \frac{2m_1m_2}{m_1+m_2}\,g = \frac{2\times 0.36\times 0.72}{1.08}\times 10$$

$T = 4.8\ N$

and acceleration of each block

$$a = \left(\frac{m_2-m_1}{m_1+m_2}\right)g = \left(\frac{0.72 = 0.36}{0.72+0.36}\right)g$$

$$a = \frac{10}{3}\ m/s^2 .$$



Let 'S' is the distance covered by block of mass 0.36 *kg* in first sec.

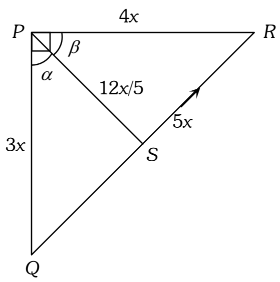
$$S = Ut + \frac{1}{2}at^2 \Rightarrow S = 0 + \frac{1}{2}\left(\frac{10}{3}\right)\times 1^2 = \frac{10}{6}\ meter$$

$\therefore$ Work done by the spring $W = T\,S = 4.8\times\frac{10}{6} \Rightarrow W = 8\ Joule$.

52. A steady current *I* goes through a wire loop *PQR* having shape of a right angle triangle with $PQ = 3x$, $PR = 4x$ and $QR = 5x$. If the magnitude of the magnetic field at *P* due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$, find the value of *k*.

Sol. Magnetic field of *P* due to *PQ* & *PR* is zero.

$\therefore$ Magnetic field at *P* due to *QR*



$$B = \frac{\mu_0}{4\pi}\cdot\frac{I}{PS}(\sin\alpha + \sin\beta)$$

where, $B = \frac{\mu_0}{4\pi}\cdot\frac{I}{\frac{12x}{5}}\left[\frac{3}{5}+\frac{4}{5}\right]$

$$B = \frac{\mu_0}{4\pi}\times\frac{I}{12x}\times 7 = \frac{7\mu_0 I}{48\pi x}$$

$\therefore\ k = 7$.

53. A solid sphere of radius *R* has a charge *Q* distributed in its volume with a charge density $\rho = kr^a$, where κ and *a* are constants and *r* is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at $r = R$, find the value of *a*.

Sol. $\rho = Kr^a$

$$E\left(r = \frac{R}{2}\right) = \frac{1}{8}E\ (r = R)$$

$$\frac{q_{enclosed}}{4\pi\varepsilon_0 (R/2)^2} = \frac{1}{8}\frac{Q}{4\pi\varepsilon_0 R^2}$$

$$32\,q_{enc.} = Q$$

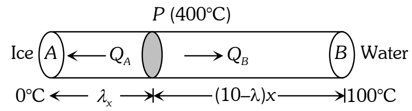
$$q_{enc.} = \int_0^{R/2} Kr^a\,4\pi r^2\,dr = \frac{(4\pi K)}{(a+3)}\left(\frac{R}{2}\right)^{a+3}$$

$$Q = \frac{4\pi K}{(a+3)}R^{a+3}$$

$$\frac{Q}{q_{enc.}} = 2^{a+3} \Rightarrow 2^{a+3} = 32 \Rightarrow a = 2 .$$

54. A metal rod AB of length $10x$ has its one end A in ice at $0°C$ and the other end B in water at $100°C$. If a point P on the rod is maintained at $400°C$, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ. [Neglect any heat loss to the surrounding].

Sol.



Heat received by end A, for melting of ice $Q_A = \dfrac{K\,A\,(400-0)\,t}{\lambda_x} = m\,L_{ice}$ .....(i)

Heat received by end B, for vaporisation of water $Q_B = \dfrac{K\,A\,(400-100)\,t}{(10-\lambda)x} = m\,L_{vap.}$ .....(ii)

Dividing both equation, $\dfrac{\frac{400}{\lambda_x}}{\frac{300}{(10-\lambda)\,x}} = \dfrac{L_{ice}}{L_{vap}} \Rightarrow \dfrac{4}{3}\dfrac{(10-\lambda)}{\lambda} = \dfrac{80}{540} \Rightarrow \lambda = 9$.

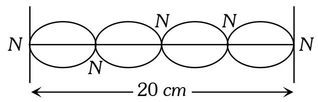
55. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation **(in *cm*)** between the successive nodes on the string.

Sol. Frequency of standing wave, $n = \dfrac{P}{2}\sqrt{\dfrac{T}{ml}}$

Substituting the given value $n = 100\ Hz,\ T = 0.5\ N,\ m = 1\times10^{-3}\ kg,\ l = 0.2\ m$

We get $P = 4$, i.e., in a string of 20 cm length four loops are present

Hence the distance between two successive nodes is 5 cm.



56. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure $8\ N/m^2$. The radii of bubbles A and B are 2 cm and 4cm, respectively. Surface tension of the soap-water used to make bubbles is $0.04\ N/m$. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity].

Sol. Excess pressure inside the soap bubble $= \dfrac{4S}{r}$

So the pressure inside the soap bubble = $P_{atm} + \dfrac{4S}{r}$

From ideal gas equation $PV = nRT$

$$\frac{P_A V_A}{P_B V_B} = \frac{n_A}{n_B} \Rightarrow \frac{\left(8 + \frac{4S}{r_A}\right)\frac{4}{3}\pi (r_A)^3}{\left(8 + \frac{4S}{r_B}\right)\frac{4}{3}\pi ((r_B)^3} = \frac{n_A}{n_B}$$

Substituting $S = 0.04\ N/m$, $r_A = 2\ cm$, $r_B = 4\ cm$.

$\dfrac{n_A}{n_B} = \dfrac{1}{6}$

$\therefore\ \dfrac{n_B}{n_A} = 6$.

57. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height **(in *mm*)** of water level due to opening of the orifice.

[Take atmospheric pressure $= 1.0\times10^5\ N/m^2$, density of water $= 1000\ kg/m^3$ and $g = 10\ m/s^2$. Neglect any effect of surface tension].

Sol. **(6)**